



ISSN: 2617-2070 (Print) ; 2617-2070 (Online)

Journal of Advanced Sciences and Engineering Technologies

Available online at: <http://www.jaset.isnra.org>

Journal of Advanced  
Sciences and Eng.  
Technologies

**JASET**

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### Keywords

Inverse Rayleigh Distribution, Stress-  
Strength Cascade system, Estimation.

### ARTICLE INFO

#### Article history:

#### Article history:

Received 01 September 2020

Accepted 01- October -2020

Available online 10- October 2020

DOI:

<http://dx.doi.org/10.32441/jaset.03.02.02>

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# Cascade stress-strength system Reliability Estimation of Inverse Rayleigh Distribution

## Abstract

The electronic devices, equipment and complex machines used in many fields such as telecommunications, medicine, astronautics and others are all subject to malfunctions, which cause material and moral losses, waste of time and other damages. Hence the importance of reliability issue in our working life by evaluating the performance and efficiency of these systems and Measuring the reliability of any device will be the basis for the development of most of these devices . Then In this paper will discussed the Estimation of Reliability  $R_n$  for cascade system when the stress and strength are Inverse Rayleigh distributed random variables. undervoltage rating.

The cascade system is a redundant component system, which is a redundant component with undervoltage rating and independently distributed power, in which the redundant component replaces the faulty component.. Cascade system is a special case of Stress-Strength models system. Also we discussed the Estimation of Marginal Reliabilities  $R_1$ ,  $R_2$  and  $R_3$  for Cascade system by three estimation methods (Max. likelihood, Weighted Least Square, Least Square) and Compare between the estimators of  $R_4$ .

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## Introduction

In general the concept of the statistical for stress-strength describes the nature of the relation between two random variables representing stress and strength is to find and estimate probability that one of the two variables overrides the other variable [2]. The stress - strength reliability refers to the life of a component with random strength when subjected to random stress. Standby redundancy system is a well-known technique for increasing the reliability of a system. Here a number of redundant components are attached to one or more essential components of the system and in this system one of these components replaces the component that failed [16]. Cascade system that is a special case of standby redundancy for stress-strength models [9], this system inside there are many advanced electronic devices and equipment, it's that were first studied by Sriwastav and Pandit [5]. In most of the studies mentioned in the literature on the cascade system, the study is only carried out taking into account the effect of the stress reduction factor, this system have m-component if the first component fails to resist the strength of the second component, it

is reactivated and faces the effect of the process pressure in sequence for only one-component at a time [6]. This observation motivated the current work to design a reliability for a cascade model under the combined effect of stress and strength attenuation factors. The reliability of the system was achieved with the help of certain density functions of the m-standby system, in which all stress-strengths are random variables [4]. The reliability phrases of an m-cascade system are obtained when the stress-strength of the m-components follow particular distributions [7]. In addition, the reliability assessment (estimation of the reliability fun.) is performed using standard methods. The paper is structured as follows. In Section 1 introduction for cascade system. In section 2 the general model is developed with the reliability expressions for cascade system is when was it received the stress strength of the components follows particular distributions. In section 3 we provided the properties of Inverse Rayleigh Distribution. In section 4 the expressions of  $R_m$  are obtained when both stress-strength are Inverse Rayleigh distribution. In section 5 Estimation of

Reliability and Marginal Reliabilities for Inverse Rayleigh distribution. In section 6 Analysis of the results of Estimation.

**Ease of Use**

**General Mode**

The reliability of the system was obtained using certain numbers for the density functions of the n-standby system, where all stress-strengths are random variables.[1]. the reliability of stress-strength system of a component is defined by  $R = P(\text{stress} < \text{strength})$  probability that the strength for the component, X is not less than the stress Y on it. Symbolically;  $R = \text{pr}(X \geq Y)$  [11], such that X and Y be random variables independent are positive representing to the strengths and stresses respectively[13], and this system totally fails if and only if the pressure of X is greater than its Y it mean the reliability of cascade system could survive with a loss of the first components (m - 1) if and only if  $X_i \leq Y_i ; i = 1,2,3, \dots, m - 1$  and  $X_m > Y_m$ .

And the System Reliability of m-components for Cascade Model  $R_s$  of the cases  $s=1,2,3,4$ [8],is:

$$R_s = \sum_{m=1}^s R(m) \dots(1)$$

; where R(m) is the marginal reliability of m-component to the reliability of the system and is defined by [6]:

$$R(s) = P[x_1 < y_1, x_1 < ky_1, \dots, 1 < k^{s-2}y_1, x_1 \geq k^{s-1}y_1] \\ x_1, y_1 \geq 0 \dots(2)$$

Where factor  $k > 1$  and which is called " stress attenuation factor "and it is the reason why the cascade system is a special type of standby system because it introduces an improvement factor for the subsequent component on the previous [7]

R(1) is the extraction of first marginal reliability of Reliability for cascade system based on one-

component only when  $x_1$  is the strength and  $y_1$  is the stress. R(2) is the extraction of the second marginal reliability of Reliability for cascade system the dependence on the second component, when the failure of the first component in the face of the stratcon effect on the first component, and  $x_2$  is the second strength affecting the second component and  $y_2 = ky_1$  is the dependence adopted here. R(3) is the extraction of third marginal reliability of Reliability for cascade system of the reliability based on the third component, when the failure of the second component in the face,  $x_3$  is the strength affected the component and  $y_3 = k^2y_1$  is the stress dependence adopted here. R(4) is the extraction of fourth marginal reliability of Reliability for cascade system based on the 3-component, after the failure of the third component in the face of the stratified component [7].

**Inverse Rayleigh Distribution**

This distribution is a special type of inverse Weibull distribution [10]. This distribution has many uses in reliability studies. Units can be approximated by the it, also this distribution is an important lifetime in survival analysis that the inverse Rayleigh distribution is introduced in (1972) by Voda. [3].

pdf of the Inverse Rayleigh distribution with scale parameter  $\theta$  is define as follows: [14]  $f(x; \theta) = \frac{2\theta}{x^3} \exp\left(-\frac{\theta}{x^2}\right); x > 0, \theta > 0 \dots (3)$

And the C.D.F is given by:

$$F(x; \theta) = \exp\left(-\frac{\theta}{x^2}\right); x > 0, \theta > 0 \dots (4)$$

Table (1) Some properties for Inverse Rayleigh Distribution [7]

Parameter	$\theta > 0$ scale (real)
Support	$x \geq 0$

PDF	$\frac{2\theta}{x^3} \exp\left(-\frac{\theta}{x^2}\right)$
CDF	$\exp\left(-\frac{\theta}{x^2}\right)$
Mean	$\sqrt{\pi}$
Medium	$1/3(\sqrt{\frac{\theta}{3/2}} + 2\sqrt{\pi})$
Mode	$\sqrt{\frac{\theta}{3/2}}$
Variance	not exist
Skewness	not exist
Kurtosis	not exist

**Reliability of Cascade stress-strength system for Inverse Rayleigh Distribution**

The pdf with random variables  $X_i ; i = 1,2, \dots, m$  be the strength with scale parameter  $\theta$  and the probability density function random variable  $y_1$  be the stress with scale parameter  $\rho$  for Inverse Rayleigh respectively given as:

$$f(x_i) = \frac{2\theta}{(x_i)^3} e^{-\frac{\theta}{(x_i)^2}} \dots(5)$$

$$g(y_1) = \frac{2\rho}{y_1^3} e^{-\frac{\rho}{y_1^2}} \dots(6)$$

and the cumulative distribution functions respectively, given as :

$$F(x_i, \theta) = e^{-\frac{\theta}{x_i^2}} \dots (7)$$

$$G(y_1, \rho) = e^{-\frac{\rho}{y_1^2}} \dots (8)$$

The Marginal Reliabilities of Inverse Rayleigh Distribution [6] are:

$$R(1) = \frac{\theta}{\theta + \rho} \dots(9)$$

$$R(2) = \frac{\rho}{k^3} \left[ \frac{1}{(\theta + \rho)} - \frac{1}{(\theta + \rho + \frac{\theta}{k^2})} \right] \dots(10)$$

$$R(3) = \rho \left[ \frac{1}{(\theta + \rho + \frac{\theta}{k^2})} - \frac{1}{(\theta + \rho + \frac{\theta}{k^2} + \frac{\theta}{k^4})} \right] \dots(11)$$

Then the total reliability  $R_m$  of the m-cascade for Inverse Rayleigh Distribution [6] are: -

$$R_1 = \frac{\theta}{\theta + \rho}$$

$$R_2 = \frac{\theta}{\theta + \rho} + \rho \left[ \frac{1}{\theta + \rho} - \frac{1}{\theta + \rho + \frac{\theta}{k^2}} \right] \dots(12)$$

$$R_3 = \frac{\theta}{\theta + \rho} + \rho \left[ \frac{1}{\theta + \rho} - \frac{1}{\theta + \rho + \frac{\theta}{k^2}} \right] + \rho \left[ \frac{1}{\theta + \rho + \frac{\theta}{k^2}} - \frac{1}{\theta + \rho + \frac{\theta}{k^2} + \frac{\theta}{k^4}} \right] \dots(13)$$

**Estimation: -**

In this sub section, the unknown scale parameter of Inverse Rayleigh and the system reliability function  $R_4$  have been estimated by four different estimation methods; (Maximum likelihood, Weighted Least Square and Least square).

**Maximum likelihood method: -**

The maximum likelihood method (ML) is proposed by R.A. Fisher (1912) [15] and has been widely used since. Let  $x_1, x_2, \dots, x_m$  strength random sample of Inverse Rayleigh with  $\theta$  unknown scale parameter and with sample size  $m$ , then The maximum likelihood estimator  $\hat{\theta}_{ML}$  of  $\theta$  that that maximizes the likelihood function is defined by using equation (3) [9]:

$$L(x_1, x_2, \dots, x_m, \theta) = (2\theta)^m \prod_{i=1}^m x_i^{-3} e^{-\sum_{i=1}^m \frac{\theta}{x_i^2}} \dots (14)$$

Then  $\ln L$  for the equation (14) can be written as:

$$\ln L = m \ln \theta + m \ln 2 - 3 \sum_{i=1}^m \ln x_i - \theta \sum_{i=1}^m \frac{1}{x_i^2} \dots (15)$$

take the partial derivative to equation(15) with the unknown parameter  $\theta$  then:

$$\frac{\partial \ln L}{\partial \theta} = \frac{m}{\theta} - \sum_{i=1}^m \frac{1}{x_i^2}$$

Equating the partial derivative to zero, then the ML estimator for  $\theta$ ,  $\hat{\theta}_{(ML)}$  is given by:

$$\hat{\theta}_{(ML)} = \frac{m}{\sum_{i=1}^m \frac{1}{x_i^2}} = \frac{m}{T} \dots (16)$$

so the Maximum likelihood estimator for  $R_3$ , say  $\hat{R}_{3(ML)}$  is given The by substituting equations (16) in equations (13), we get:

$$R_3 = \frac{\hat{\theta}_{(ML)}}{\hat{\theta}_{(ML)} + \rho} + \rho \left[ \frac{1}{\hat{\theta}_{(ML)} + \rho} - \frac{1}{\hat{\theta}_{(ML)} + \rho + \frac{\hat{\theta}_{(ML)}}{k^2}} \right] + \rho \left[ \frac{1}{\hat{\theta}_{(ML)} + \rho + \frac{\hat{\theta}_{(ML)}}{k^2}} - \frac{1}{\hat{\theta}_{(ML)} + \rho + \frac{\hat{\theta}_{(ML)}}{k^2} + \frac{\theta}{k^4}} \right] \dots (17)$$

**Least Square Method: -**

The least square method (LS) can be produced by minimizing the sum of square error between the value and its expected value [3]. This method is a combination of the parametric (F) and the non-parametric ( $\hat{F}$ ) distribution functions .The minimizing following equation : [2]

$$S = \sum_{i=1}^n \left( \hat{F}(x_i) - F(x_i) \right)^2 \dots (19)$$

Suppose that  $x_i$   $i= 1,2,\dots,m$  be a random sample have  $INR(\theta)$  distribution with the sample size  $m$ . The procedure attempts to minimize will get as:

$$S(\theta) = \sum_{i=1}^m \left( \hat{F}(x_i) - \left( e^{-\frac{\theta}{x_i^2}} \right) \right)^2 \dots (20)$$

To obtain the formula of  $F(x_i)$  ; use the equation (4) :

$$F(x_i) = e^{-\frac{\theta}{x_i^2}} \dots (21)$$

Now taking the the  $\ln$  of the equation (21), we get:

$$-\ln(F(x_i)) = \frac{\theta}{x_i^2}$$

On the other hand, since  $\hat{F}(x_i)$  is unknown , it better to use  $\hat{F}(x_{(i)})$  as follows

$$\hat{F}(x_{(i)}) = P_i \text{ and } P_i \text{ is the plotting position}$$

$$\text{Where } P_i = \frac{i}{m+1} ; i = 1,2,\dots, m$$

Where  $i$ : the order statistics of the random sample, to obtain the LS estimates  $\hat{\theta}_{(LS)}$  of the parameter  $\theta$  can be define following the function from equation (20) :

$$S(\theta) = \sum_{i=1}^n \left( q_i - \frac{\theta}{x_{(i)}^2} \right)^2 \dots (22)$$

$$\text{Where } q_i = -\ln \hat{F}(x_{(i)}) = -\ln P_i$$

By taking the derivative equation (22) with the parameter  $\theta$  and equal the result to the zero :

$$\frac{\partial S(\theta)}{\partial \theta} = \sum_{i=1}^n 2 \left( q_i - \frac{\theta}{x_{(i)}^2} \right) \left( -\frac{1}{x_{(i)}^2} \right) - \sum_{i=1}^n \frac{q_i}{x_{(i)}^2} + \hat{\theta} \sum_{i=1}^n \frac{1}{(x_{(i)}^2)^2} = 0 \dots (23)$$

By simplification the equation (23), we get:

$$\hat{\theta}_{(LS)} = \frac{\sum_{i=1}^n \frac{q_i}{x_{(i)}^2}}{\sum_{i=1}^n \frac{1}{(x_{(i)}^2)^2}} \dots (24)$$

so the Least Square estimator for  $R_3$ , say  $\hat{R}_{3(LS)}$  is given The by substituting equations (24) in equations (13), we get:

$$\hat{R}_{3(LS)} = \frac{\hat{\theta}_{(LS)}}{\hat{\theta}_{(LS)} + \rho} + \rho \left[ \frac{1}{\hat{\theta}_{(LS)} + \rho} - \frac{1}{\hat{\theta}_{(LS)} + \rho + \frac{\hat{\theta}_{(LS)}}{k^2}} \right] + \rho \left[ \frac{1}{\hat{\theta}_{(LS)} + \rho + \frac{\hat{\theta}_{(LS)}}{k^2}} - \frac{1}{\hat{\theta}_{(LS)} + \rho + \frac{\hat{\theta}_{(LS)}}{k^2} + \frac{\theta}{k^4}} \right] \dots (25)$$

**Weighted Least Square Method :-**

The weighted least squares method (WLS) obtain by minimizing following equation [8], with respect to the unknown parameter  $\theta$  of strength random variable  $x_i \sim INR(\theta)$ .

$$Q = \sum_{i=1}^m w_i \left( \hat{F}(x_i) - F(x_i) \right)^2 \dots (26)$$

Where  $w_i = \frac{1}{\text{Var}[F(x_i)]} = \frac{(m+1)^2(m+2)}{i(m-i+1)}, i = 1, 2, \dots, m$

suppose  $x_{(i)}, i = 1, 2, \dots, m$  be random variable of Inverse Rayleigh with sample size  $m$ .

The procedure attempts to minimize the following function with respect to  $\alpha$  and  $\beta$  will get as :

$$Q(\theta) = \sum_{i=1}^m w_i \left( \hat{F}(x_i) - e^{-\frac{\theta}{x_{(i)}^2}} \right)^2 \dots (27)$$

$$Q(\theta) = \sum_{i=1}^m w_i \left( q_i - \frac{\theta}{x_{(i)}^2} \right)^2 \dots (28)$$

Where  $q_i = -\ln \hat{F}(x_{(i)}) = -\ln P_i$

By taking partial derivative to the equation (28) with parameter  $\theta$ , and equating result to the zero we obtain:

$$\sum_{i=1}^m 2 w_i \left[ q_i - \frac{\theta}{x_{(i)}^2} \right] \frac{-1}{x_{(i)}^2}$$

$$\sum_{i=1}^m \frac{-w_i}{x_{(i)}^2} q_i + \hat{\theta} \sum_{i=1}^m \frac{w_i}{x_{(i)}^4} = 0 \dots (29)$$

Then, by simplification the equation (29), we get:

$$\hat{\theta}_{(WLS)} = \frac{\sum_{i=1}^m \frac{w_i}{x_{(i)}^2}}{\sum_{i=1}^m \frac{w_i}{x_{(i)}^4}} q_i \dots (30)$$

then  $\hat{R}_{3(WLS)}$  is given by substituting equations (30) in equation (13) we get:

$$\hat{R}_{4(WLS)} = \left[ \frac{1}{\hat{\theta}_{(WLS)} + \rho + \frac{\hat{\theta}_{(WLS)}}{k^2}} \right] + \rho \left[ \frac{1}{\hat{\theta}_{(WLS)} + \rho + \frac{\hat{\theta}_{(WLS)}}{k^2}} - \frac{1}{\hat{\theta}_{(WLS)} + \rho + \frac{\hat{\theta}_{(WLS)}}{k^2} + \frac{\hat{\theta}_{(WLS)}}{k^4}} \right] \dots (32)$$

**Analysis of the results of Estimation: -**

Since the mean results of  $R_3$  in all the tables below are nearest from  $R$ -real then the results are acceptable

Table (2): Results of Mean, MSE values when (m=15)

$\theta, \rho, k$	$R_3$	WLS	LS	ML	M=15	Best
<b>(0.6,0.7,6)</b>	0.4685	0.4009 0.0046	0.9596 0.2411	0.3143 0.0238	Mean MSE	- <b>WLS</b>
<b>(0.5,0.7,5)</b>	0.4266	0.2698 0.0246	0.3518 0.0056	0.3002 0.0160	Mean MSE	- <b>LS</b>
<b>(0.4,0.7,4)</b>	0.3787	0.4733 0.0090	0.9154 0.2880	0.4052 0.0007	Mean MSE	- <b>ML</b>
<b>(0.3,0.7,3)</b>	0.3253	0.1510 0.0304	0.0018 0.1046	0.2041 0.0147	Mean MSE	- <b>ML</b>
<b>(0.2,0.7,2)</b>	0.2751	0.3231 0.0023	0.9033 0.3947	0.3785 0.0107	Mean MSE	- <b>WLS</b>

<b>(0.1,0.7,1)</b>	0.3636	0.4800	0.7328	0.3762	Mean MSE	- <b>ML</b>
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Table (3): Results of Mean, MSE values when (m=25)

$\theta, \rho, k$	<b>R<sub>3</sub></b>	<b>WLS</b>	<b>LS</b>	<b>ML</b>	<b>m=25</b>	<b>Best</b>
<b>(0.6,0.7,6)</b>	0.4685	0.6002 0.0173	0.2195 0.0620	0.5356 0.0045	Mean MSE	- <b>ML</b>
<b>0.5,0.7,5</b>	0.4266	0.3295 0.0094	0.0798 0.1203	0.4753 0.0024	Mean MSE	- <b>ML</b>
<b>0.4,0.7,4</b>						- <b>ML</b>
<b>0.3,0.7,3</b>	0.3787	0.4693 0.0082	0.2072 0.0294	0.4129 0.0012	Mean MSE	- <b>ML</b>
<b>0.2,0.7,2</b>	0.3253	0.1952	0.1576	0.2439	Mean MSE	- <b>WLS</b>
<b>0.1,0.7,1</b>		0.0169	0.0281	0.0066		- <b>ML</b>

Table (4) : Results of Mean, MSE values when (m=70)

$(\theta, \rho, k)$	<b>R<sub>3</sub></b>	<b>WLS</b>	<b>LS</b>	<b>ML</b>	<b>m=70</b>	<b>Best</b>
<b>(0.6,0.7,6)</b>	0.4685	0.3656 0.0106	0.2676 0.0404	0.4070 0.0038	Mean MSE	- <b>ML</b>
<b>(0.5,0.7,5)</b>	0.4266	0.3441 0.0068	0.9730 0.2985	0.4213 0.0000	Mean MSE	- <b>ML</b>
<b>(0.4,0.7,4)</b>	0.3787	0.3186 0.0036	0.0593 0.1020	0.3267 0.0027	Mean MSE	- <b>ML</b>
<b>(0.3,0.7,3)</b>	0.3253	0.3462 0.0004	0.7425 0.1740	0.3086 0.0003	Mean MSE	- <b>ML</b>
<b>(0.2,0.7,2)</b>	0.2751	0.2108 0.0041	0.0277 0.0612	0.2421 0.0011	Mean MSE	- <b>ML</b>
<b>(0.1,0.7,1)</b>						- <b>ML,SL</b>

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Table (5) : Results of Mean, MSE values when (m=90)

$(\theta, \rho, k)$	m=90	ML	LS	WLS	$R_3$	Best
<b>(0.6,0.7,6)</b>	Mean	0.4131	0.1268	0.3478	0.4685	-
	MSE	0.0031	0.1168	0.0146		<b>ML</b>
<b>(0.5,0.7,5)</b>	Mean	0.4469	0.0656	0.4066	0.4266	-
	MSE	0.0004	0.1303	0.0004		<b>ML,WLS</b>
<b>(0.4,0.7,4)</b>	Mean	0.3238	0.9915	0.2822	0.3787	-
	MSE	0.0030	0.3756	0.0093		<b>ML</b>
<b>(0.3,0.7,3)</b>	Mean	0.3158	0.3295	0.2928	0.3253	-
	MSE	0.0001	0.0000	0.0011		<b>LS</b>
<b>(0.2,0.7,2)</b>	Mean	0.2685	0.0245	0.2409	0.2751	-
	MSE	0.0000	0.0628	0.0012		<b>ML</b>
<b>(0.1,0.7,1)</b>	Mean	0.3317	0.7165	0.3435	0.3636	-
	MSE	0.0010	0.1245	0.0004		<b>WLS</b>

From the tables above, we have observed that:

**If sample size m is increasing from 15 to 90, we observed:**

- 1- From tables (2), (3), (4) and (5) which present the simulation results by Mean for estimating  $R_3$  for Cascade system with different cases, it appears that:
  - ❖ When  $(\theta, \rho, k) = (0.6,0.7,6)$ 
    - The MSE values of ML is decreasing but LS and WLS are increasing by increasing sample size.
    - When  $(\theta, \rho, k) = (0.5,0.7,5)$
    - The MSE values of ML and WLS are decreasing but LS is increasing by increasing sample size
  - ❖ When  $(\theta, \rho, k) = (0.4,0.7,4)$ 
    - The MSE values of ML, LS and WLS are decreasing by increasing sample size.
  - ❖ When  $(\theta, \rho, k) = (0.3,0.7,3)$

- The MSE values of ML, LS and WLS are decreasing by increasing sample size

- ❖ When  $(\theta, \rho, k) = (0.3,0.7,3)$ 
  - The MSE values of MO, LS, RG and WLS are decreasing by increasing samples sizes.

**B. If n is constant:**

- 1- When n is small sample size (n=15), from table (2) we observe that:
  - ❖ When  $(\theta, \rho, k) = (0.6,0.7,6), (0.5,0.7,5), (0.4,0.7,4), (0.3,0.7,3), (0.2,0.7,2), (0.1,0.7,1)$
  - From above getting MSE values of ML and LS are decreasing but WLS is increasing by increasing the values of parameters  $\theta$ , and  $k$ .
- 2- When n is small sample size (n=25), from table (3) we observe that:
  - ❖ When  $(\theta, \rho, k) = (0.6,0.7,6), (0.5,0.7,5), (0.4,0.7,4), (0.3,0.7,3), (0.2,0.7,2), (0.1,0.7,1)$



From above getting MSE values of ML and LS are decreasing but WLS is increasing by increasing the values of parameters  $\theta$ , and  $k$ .

3- When  $m$  is large sample size ( $m=70$ ), from table (3) we observe that:

- ❖ When  $(\theta, \rho, k) = (0.6, 0.7, 6), (0.5, 0.7, 5), (0.4, 0.7, 4), (0.3, 0.7, 3), (0.2, 0.7, 2), (0.1, 0.7, 1)$

From above getting MSE values of ML, LS and WLS are decreasing by increasing the values of parameters  $\theta$ , and  $k$ .

4- When  $m$  is large sample size ( $m=90$ ), from table (5) we observe that:

- ❖ When  $(\theta, \rho, k) = (0.6, 0.7, 6), (0.5, 0.7, 5), (0.4, 0.7, 4), (0.3, 0.7, 3), (0.2, 0.7, 2), (0.1, 0.7, 1)$

From above getting MSE values of ML and WLS are decreasing but LS is increasing by increasing the values of parameters  $\theta$ , and  $k$ .

**Conclusions to estimated for Inverse Rayleigh :-**

- The performance of MLE is the best as in the table below.

Sample size and parameters value	Best method
For MSE when $m=15, m=25, m=70, m=90$ and $\rho = 0.7$ the MSE of ML is best in table (2),(3),(4) and(5) except when the parameter $(\theta, \rho, k)$ for $(0.6, 0.7, 6), (0.5, 0.7, 5), (0.2, 0.7, 2), (0.2, 0.7, 2), (0.3, 0.7, 3), (0.1, 0.7, 1)$ .	ML

**Table (6):**The first best method estimator of MSE for Inverse Rayleigh distribution of  $R_4$ .

**Recommendations :-**

1. The study of the Reliability of Cascade stress-strength system to include other recent distributions.
2. In the case of an assessment for Cascade system reliability by using Inverse Rayleigh distribution We recommend to use method ML .
3. From the observation of the behavior of Cascade system of this distribution , we recommend that attention be paid to the values of strength parameter.  
Extending the study of the Cascade stress-strength system Reliability assessments to include inverted Rayleigh and Lomax distributions to find the best methods that give the best estimated value .

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